

Hypothesis Testing

Example: Average intake in children for dietary fat is 70 g of fat per day. Suppose we want to study children who eat a vegetarian diet. Possible hypotheses are

1. Average fat intake is 70 g per day
2. Average fat intake is less than 70 g per day

Def: One-sample problem—a single distribution.

Def: Hypothesis—statement about parameters in a population or populations. We want to know how likely this is to be true, given the evidence (data). For example,

1. Average number of beds filled per day in the hospital
2. Average number of minutes per day the doctor spends with a patient
3. Average lead content of water for a housing project

Def: Null hypothesis— H_0 —the hypothesis to be tested. **This is usually a statement of no difference.** The population value of the parameter is not different from some specified value.

Def: Alternative hypothesis— H_1 or H_A —This is the statement we will accept if we reject the null hypothesis.

H_0 : Mean fat intake in vegetarian children is 70 g per day. $H_0: \mu = \mu_0$ or $\mu \geq \mu_0$

H_1 : Mean fat intake in vegetarian children is < 70 g per day. $H_1: \mu < \mu_0$

Possible decisions:

1. Accept H_0 (really, fail to reject H_0)
2. Reject H_0

Possible Scenarios:

Jury Trial		
	Truth	
Verdict	Innocent	Guilty
Innocent	Correct decision	Error
Guilty	Error	Correct decision

Test of hypothesis		
	Truth	
Results of Test	H₀	H₁
Accept H₀	Correct decision	Type II error
Reject H₀	Type 1 error	Correct decision

Def: Type I error is the probability of rejecting H₀ when H₀ is true.

Def: Type II error is the probability of accepting H₀ when H₁ is true.

Example: We have developed a new procedure to improve survival of premature infants. If the hospital adopts these procedures, there will have to be new rooms and new equipment purchased. This is very costly.

- 1. What does a Type I error imply?**
- 2. What does a Type II error implies?**

Def: Level of significance: α = Probability of a Type I error. This is the area under the curve below (or above) the critical value. This is the probability of rejecting H_0 when H_0 is true.

Def: β : Probability of a Type II error.

Def: $1-\beta$ —Power of a test. This is the Pr (rejecting $H_0|H_1$ is true).

Goal: Make α, β as small as possible. Usually, as $\alpha \uparrow, \beta \downarrow$ and as $\alpha \downarrow, \beta \uparrow$.

Fix α (0.05 or 0.01). Find a test to minimize β .

Best test for the fat experiment is one based on \bar{X} .

Def: Acceptance Region—These are the values of \bar{X} for which H_0 is accepted.

Def: Rejection Region—These are the values of \bar{X} for which H_0 is rejected.

Note: For this example, we are conducting a one-sided or one-tailed test. We will only reject H_0 for values of \bar{X} that are low.

Def: One-tailed—this is a test in which values of parameter under H_1 either $>$ or $<$ values under H_0 but not both.

The picture behind the two sided test

