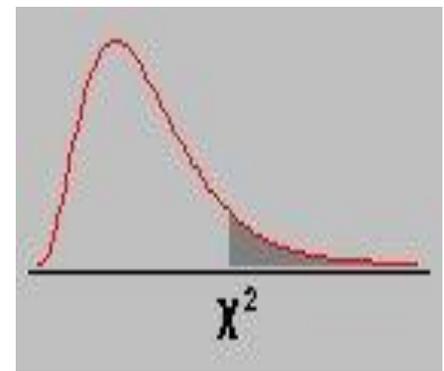


بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



السلام عليكم ورحمة الله وبركاته



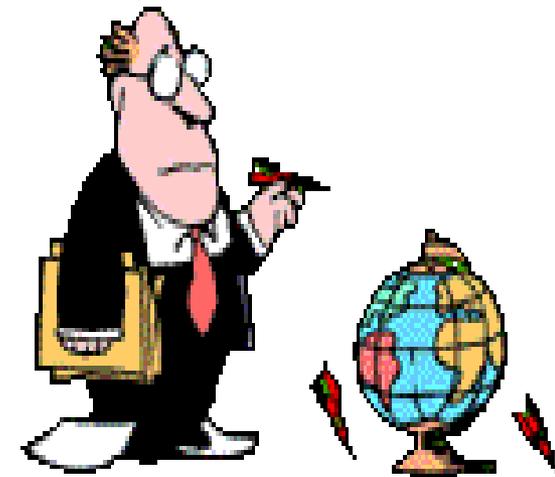
LVII

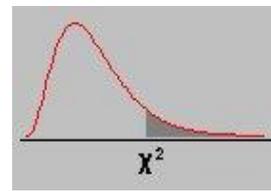
Chi Square (χ^2) test

Part 1

Prof. Dr. Waqar AL-Kubaisy

@ August 5- 2025





SPECIFIC LEARNING OUTCOMES

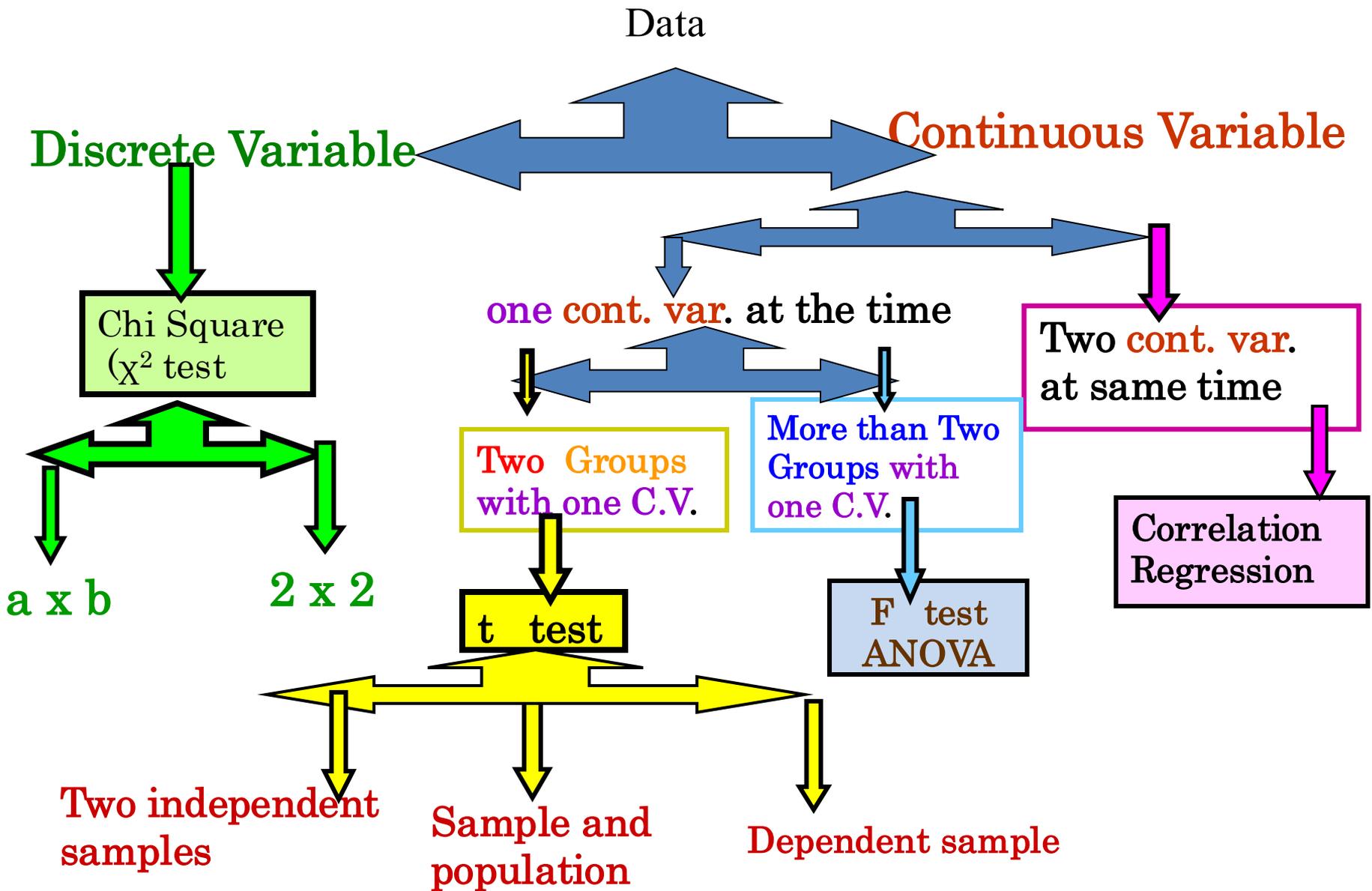
On completion of this lecture, you should be able to:

1. Explain the basis for the use of Chi square tests
2. Explain the **limitations of the Chi square** tests
3. **Carry out the** Chi square tests
4. **Interpret the findings** from the Chi square tests of significance
5. Interpret degrees **of freedom and critical** values of Chi square statistics from **Chi square table**

CONTENTS

1. **Explanation of the basis for** the use of Chi square tests on **qualitative data**

1. Explanation of the limitations of the Chi square tests
2. Calculation of Chi square
3. Chi square table
4. Interpretation of the findings from the Chi square tests of significance



An important thing is the type of the variable concerned.

when the data measurement is continuous

t test be applied

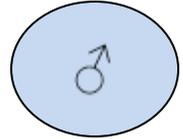
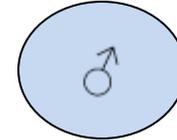
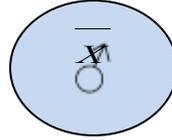
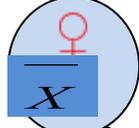
to test significance difference between **two means**

Body weight,

ANOVA (F test) be applied

to test significance difference among **more than two**

means **Body weight adult males**



Jordan,

Iraq



Palestine



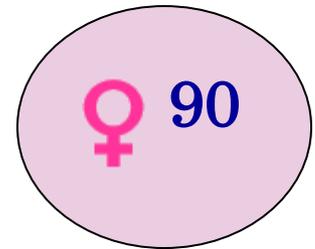
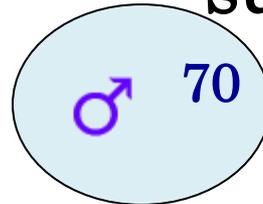
Jordan,



Iraq

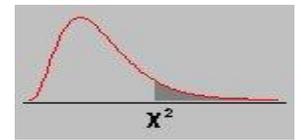
Numbers of students who were succeeded

succeeded



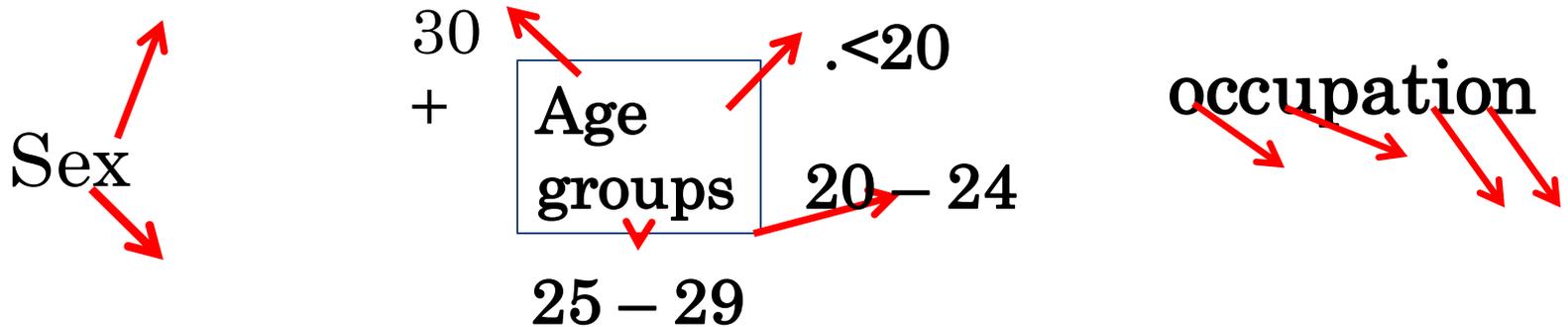
An important thing is the type of the variable concerned.





The data we have here is only **enumerative** data or **counting data** .

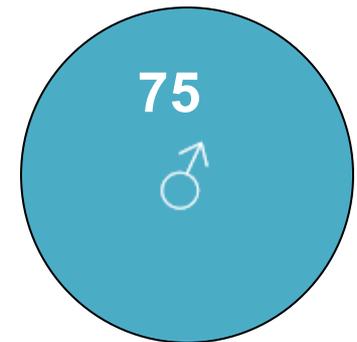
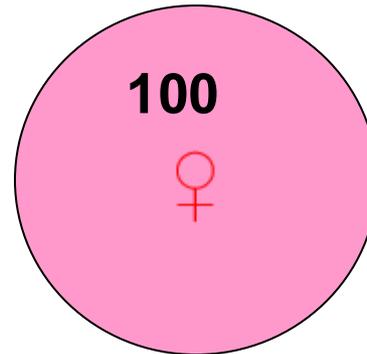
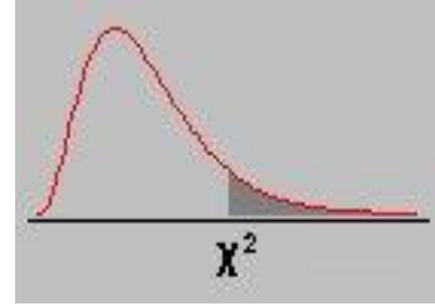
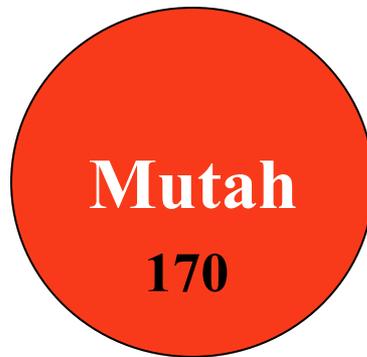
Counting No. of individuals falling in one category, class, group or another



The data consist of **counting No.** in each sample or group

An important thing is the type of the variable concerned.



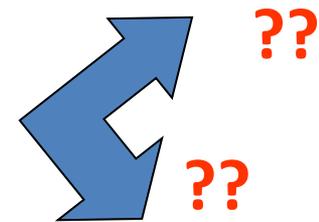


Numbers of students who were succeeded

??????

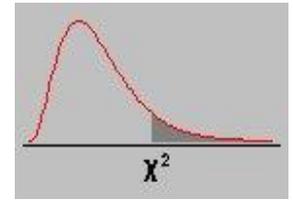
????????

cause could be



succeeded

| | |
|---------|-----|
| Baghdad | 180 |
| Mutah | 170 |



????? cause could be



succeeded

| | |
|---------|-----|
| Baghdad | 180 |
| UiTM | 220 |
| Syria | 200 |
| Mutah | 170 |

?????

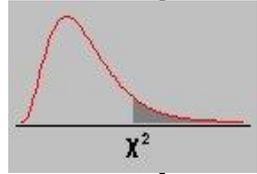


Numbers of students who were succeeded

cause could be



| | <u>Total</u> | <u>succeeded</u> | <u>%</u> | <u>Not succeeded</u> |
|---------|--------------|------------------|------------|----------------------|
| Baghdad | 240 | 180 | 75% | 60 |
| Mutah | <u>200</u> | <u>170</u> | <u>85%</u> | <u>30</u> |
| | 440 | 350 | | 90 |



Proportion succeeded

$$350/440=0.80$$

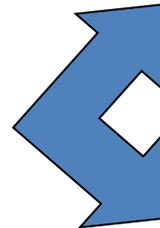
Proportion succeeded
at Mutah ??

$$0.8 \times 200 = 160$$

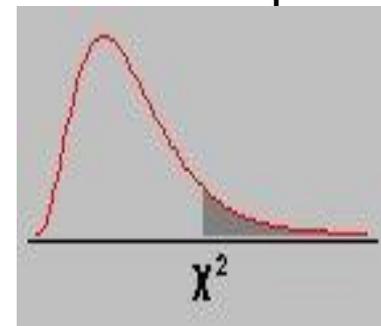
Proportion succeeded
at Baghdad ??

$$0.8 \times 240 = 192$$

cause could be



| | <u>Total</u> | <u>succeeded</u> | <u>%</u> | <u>Not succeeded</u> |
|---------|--------------|------------------|----------|----------------------|
| Baghdad | 220 | 180 | 82% | 40 |
| Mutah | 200 | 170 | 85% | 30 |
| Syria | 320 | 200 | 62.5% | 120 |
| UiTM | 380 | 220 | 57.9% | 160 |
| | 1120 | 770 | | 350 |



$$770/1120 = 0.687$$

$$350/1120 = 0.3125$$

$$770/1120 \times 100 = 68.7\%$$

$$350/1120 \times 100 = 31.25\%$$

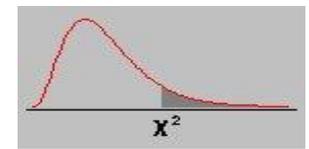
Proportion succeeded at Mutah ??

Proportion succeeded at Baghdad ??

Proportion succeeded at Syria ??

Proportion succeeded at UiTM ??

When data measurement is



counting data
Categorical data
Discrete.

The data consist of **proportion** of individuals in each group or sample

❖ We have absolute numbers

❖ We have counting numbers

❑ **comparing** between

❑ **Rates**, **proportions** of individuals in each group

❖ **Two groups**

❖ **More than two groups**

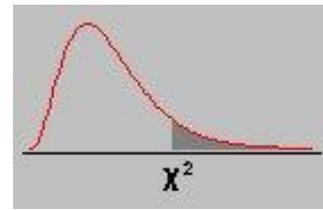
statistical inference are made
in term of **difference in proportions**



$$H_0 = P_1 = P_2 = P_0$$
$$H_A = P_1 \neq P_2 \neq P_0$$



We classify persons **into categories** such as



- male female
- Smoker not smoker
- Succeeded and not succeeded...etc
- smoker, not smoker and X smoker
- then

| | male | female | total |
|---------|------|--------|-------|
| Present | | | |
| Absent | | | |
| total | | | |

➤ count the number of observation fall in each category

The result is **frequency data**

enumerative data because we
enumerate the No. of person in each category

Categorical data , because we
count the No. of person in each category

An important thing is the type of the variable concerned.



Comparing the difference When measurement is merely the presence or absence of certain condition,

Absolute No X

✓ Proportion ✓

The Population Parameter is

P: the proportion of condition in population which is estimated by

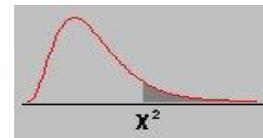
P: the proportion of condition in the sample

So

Testing hypothesis about population proportion "P" based on sample proportion P is similar to testing hypothesis about μ .

An important thing is the type of the variable concerned.





The techniques for testing hypothesis concerning
counting data
Categorical data
Discrete

is known as
chi square (χ^2) test .

Chi square is

used in testing difference in proportions

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$H_0 = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$



while t test and F test are used in testing
difference in means .

An important thing is the type of the variable concerned.



Also classification could be **more than 2 groups**, could be three, four, five K groups .

P1 P2 P3 P4 P5 Pk

Tumour stage I II III

Class stage level I II III IV V

P1 P2 P3 P4 P5 Pk

In this case

$$H_0 = P_1 = P_2 = P_3 = P_4 = P_5 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5 \neq P_0$$

| | Jordanian | Iraqi | Syrian | Egyptian | total |
|------------|-----------|-------|--------|----------|-------|
| smoker | | | | | |
| Not smoker | | | | | |
| total | | | | | |

When measurement is

merely the **presence** or absence of certain condition,

Absolute No **X**

✓ Proportion

the population parameter is

P: :the **proportion** of condition in **population**
which is estimated by

P: the **proportion** of condition in the **sample**

So

Testing hypothesis about **population proportion "P"**

based on sample proportion **P**

If the true **population proportion** of condition is **Po**

and sample size is N, So

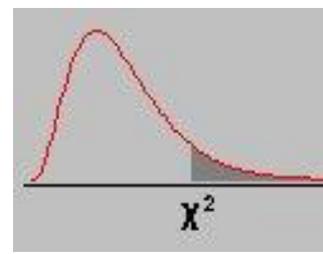
Po N = Total No. of condition that expected (**E**)

in population .

Proportion succeeded
at Mutah = $0.8 \times 200 = 160$

Proportion succeeded
at Baghdad = $0.8 \times 240 = 192$

192 = 160 ????



| | <u>Total</u> | <u>succeeded</u> | <u>%</u> | <u>Not succeeded</u> |
|---------|--------------|------------------|----------|----------------------|
| Baghdad | 240 | 180 | 75% | 60 |
| Mutah | 200 | 170 | 85% | 30 |
| | 440 | 350 | | 90 |

Proportion succeeded
 $350/440 = 0.80$

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

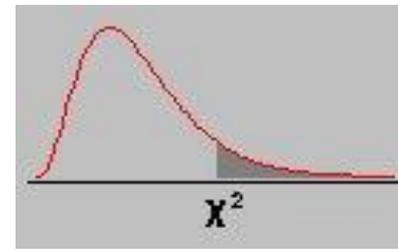
Chi square test denoted χ^2

This has two common applications:

first as test

whether **two** categorical **variables** are independent or not;

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



second as a test of

whether two **proportions** are **equal** or not

$$H_0 = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

$$H_0 = P_1 = P_2 = P_3 = P_4 = P_5 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5 \neq P_0$$

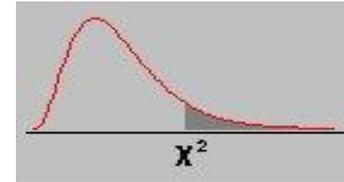


❑ **The chi square** test is applied to **frequency** data in form of a **contingency table**

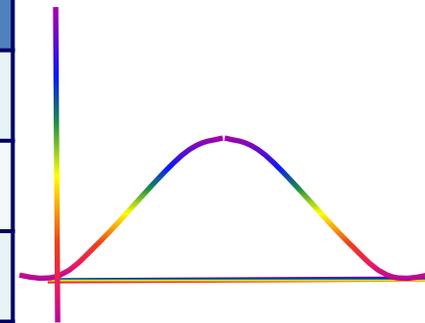
i.e. a table of cross-tabulations) with

❖ **the rows** represent categories of **one variable** and

❖ **the columns** categories of a **second variable**.



| | ♂ | ♀ | total |
|---------------|-----------|----------|-------|
| succeeded | 70(87.5%) | 90 (75%) | 160 |
| not succeeded | 10 () | 30 () | 40 |
| Total | 80 | 120 | 200 |



❑ **The null hypothesis**

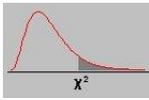
a) is that the **two variables** are **unrelated**

b) **??????????**

the **rows** represent categories of **one variable** and
the **columns** categories of a **second variable**

| Sex | succeeded | not succeeded | Total |
|-------|------------|---------------|-------|
| ♂ | 70 (87.5%) | 10 | 80 |
| ♀ | 90 (75%) | 30 | 120 |
| Total | 160 | 40 | 200 |

The H₀; is that the **two variables are unrelated**
The H_A **????????????????**



If the variables display are Exposure and outcome.

Then

usually we arrange the table with

Exposure as the **row** variable and

Out come as the **column** variable .

and display % corresponding the exposure variable

| Exposure | Out come +ve | Out come -ve | total |
|----------|--------------|--------------|-------|
| yes | | | |
| no | | | |
| Total | | | |

Example

smoking during pregnancy and relation to **small birth weight**

smoker or non smoked mother during pregnancy??

small birth weight no small birth weight ???



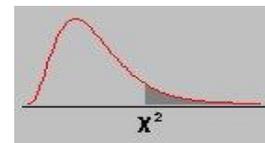
| | Small birth weight | no small birth weight | Total |
|------------------------------------|--------------------|-----------------------|-------|
| smoker mother during pregnancy | | | |
| non smoked mother during pregnancy | | | |
| Total | | | |

merely the **presence** or **absence** of certain condition,

Absolute No X

✓ **Proportion**

An important thing is the type of the variable concerned.



| | ♂ | ♀ | total |
|---------------|----------|--------|---------|
| succeeded | 70 87.5% | 90 75% | 160 80% |
| not succeeded | 10 12.5% | 30 25% | 40 |
| Total | 80 | 120 | 200 |

If the true population proportion of condition is
 $160/200 = 0.8$ $40/200 = 0.2$

$P_0 = 0.8$ and

Rate (proportion) of succeeded ♂ (p_1) = $70/80 = 87.5\%$

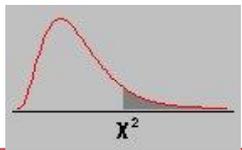
Rate (proportion) of succeeded ♀ (p_2) = $90/120 = 75\%$

$$H_0 = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

?????





| | ♂ | ♀ | total |
|---------------|------------|----------|---------|
| succeeded | 70 (87.5%) | 90 (75%) | 160 80% |
| not succeeded | 10 (12.5%) | 30 (25%) | 40 |
| Total | 80 | 120 | 200 |

If the true **population proportion** of condition is $160/200 = 0.8$ and $40/200 = 0.2$

$P_o = 0.8$ and sample size is N , (200) So

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$P_o N =$ Total No. of condition that **expected (E)** in **Each population** .

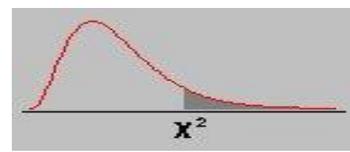
♂ $80 \times 0.8 = 64$ $80 \times 0.2 =$
 ♀ $120 \times 0.8 = 96$ $120 \times 0.2 =$

sign. Difference in proportion



expected (E)

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



| | | |
|---|---------|---------|
| ♂ | 80X.8= | 80X.2 = |
| ♀ | 120X.8= | 120X.2= |

| Out come | ♂ | | ♀ | | total |
|---------------|----|----|-----|----|-------|
| | O | E | O | E | |
| succeeded | 70 | 64 | 90 | 96 | 160 |
| not succeeded | 10 | 16 | 30 | 24 | 40 |
| Total | 80 | | 120 | | 200 |

$$\sum O - E = Zero$$

$$\sum \frac{O - E}{E} = Zero$$

- ❖ the actual **observed** No. of subject with condition (**O**)
- ❖ and the **expected** No. of condition (**E**)
- ❖ Looking for the **difference** between the
- ❖ **observed** and **expected** frequencies

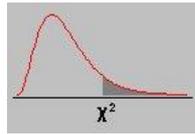
$$\sum O - E = Zero$$

$$\sum \frac{O - E}{E} = Zero$$



So if the actual No. of subject with condition observed No. (**O**) is close to the expected No. (**E**) then the H_0 will be not rejected ().

This mean that $P=P_o$.



Usually summation

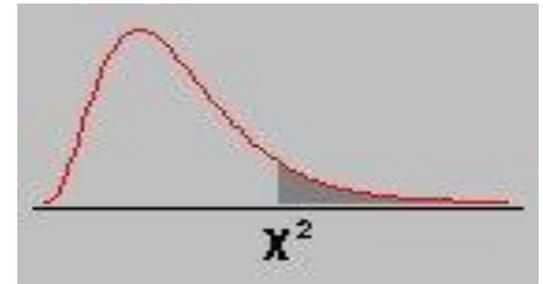
$$\sum O - E = Zero \quad \sum \frac{O - E}{E} = Zero \quad \text{So}$$

To overcome this result, we have to square $O-E$ make it as $(O-E)^2$ then divided by E for each cell $\frac{(O - E)^2}{E}$

Then we have to do the summation $\chi^2 = \sum \frac{(O - E)^2}{E_{25}}$

Therefore, χ^2 is always **UPPER ONE SIDED TEST**

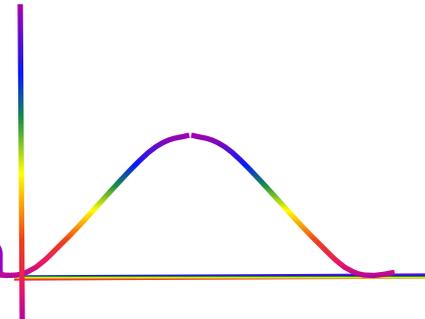
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



sign. Difference in proportion

Therefore, χ^2 is always **UPPER ONE SIDED TEST**

Comparing **calculated** χ^2 with **tabulated** χ^2 in relation to **critical region**



Critical region;

❖ Level of significance 0.95, $\alpha = 0.05$

❖ **d.F = (No. of rows - 1) (No. of column - 1)**

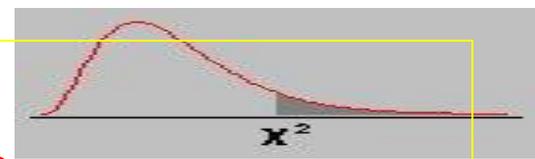
$$= (r - 1) (c - 1)$$

$$(2 - 1) (2 - 1) = 1$$

| | male | female | total |
|---------|------|--------|-------|
| Present | | | |
| Absent | | | |
| total | | | |

Chi square (χ^2)

It is the **sum** of the **squared difference** between the **observed** frequency and **expected** frequency, divided by the **expected** frequency .



$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Chi square is

used in testing **difference in proportions**

while t test and F test are used in testing difference in means .

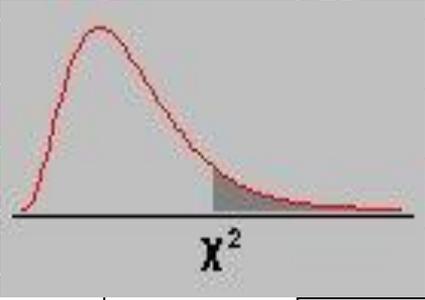
$$H_0 = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$

sign. Difference in proportion

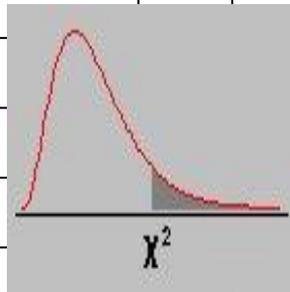
Table of Chi-square statistics

| df | P=0.05 | P= 0.01 | P= 0.001 |
|----|--------|---------|----------|
| 1 | 3.84 | 6.64 | 10.83 |
| 2 | 5.99 | 9.21 | 13.82 |
| 3 | 7.82 | 11.35 | 16.27 |
| 4 | 9.49 | 13.28 | 18.47 |
| 5 | 11.07 | 15.09 | 20.52 |
| 6 | 12.59 | 16.81 | 22.46 |
| 7 | 14.07 | 18.48 | 24.32 |
| 8 | 15.51 | 20.09 | 26.13 |
| 9 | 16.92 | 21.67 | 27.88 |
| 10 | 18.31 | 23.21 | 29.59 |
| 11 | 19.68 | 24.73 | 31.26 |
| 12 | 21.03 | 26.22 | 32.91 |
| 13 | 22.36 | 27.69 | 34.53 |
| 14 | 23.69 | 29.14 | 36.12 |
| 15 | 25.00 | 30.58 | 37.70 |
| 16 | 26.30 | 32.00 | 39.25 |
| 17 | 27.59 | 33.41 | 40.79 |
| 18 | 28.87 | 34.81 | 42.31 |
| 19 | 30.14 | 36.19 | 43.82 |
| 20 | 31.41 | 37.57 | 45.32 |
| 21 | | | |
| 22 | | | |
| 23 | | | |
| 24 | | | |
| 25 | | | |
| 26 | | | |
| 27 | | | |
| 28 | | | |
| 29 | | | |
| 30 | | | |
| 31 | | | |
| 32 | | | |
| 33 | | | |
| 34 | | | |
| 35 | | | |
| 36 | | | |
| 37 | | | |
| 38 | | | |
| 39 | | | |
| 40 | | | |



| | | | | |
|----|-------|-------|-------|-------|
| 21 | | 32.67 | 38.93 | 46.80 |
| 22 | | 33.92 | 40.29 | 48.27 |
| 23 | | 35.17 | 41.64 | 49.73 |
| 24 | | 36.42 | 42.98 | 51.18 |
| 25 | | 37.65 | 44.31 | 52.62 |
| 26 | | 38.89 | 45.64 | 54.05 |
| 27 | | 40.11 | 46.96 | 55.48 |
| 28 | | 41.34 | 48.28 | 56.89 |
| 29 | | 42.56 | 49.59 | 58.30 |
| 30 | | 43.77 | 50.89 | 59.70 |
| 31 | | 44.99 | 52.19 | 61.10 |
| 32 | | 46.19 | 53.49 | 62.49 |
| 33 | | 47.40 | 54.78 | 63.87 |
| 34 | | 48.60 | 56.06 | 65.25 |
| 35 | | 49.80 | 57.34 | 66.62 |
| 36 | | 51.00 | 58.62 | 67.99 |
| 37 | | 52.19 | 59.89 | 69.35 |
| 38 | | 53.38 | 61.16 | 70.71 |
| 39 | 55.76 | 54.57 | 62.43 | 72.06 |
| 40 | | | | |

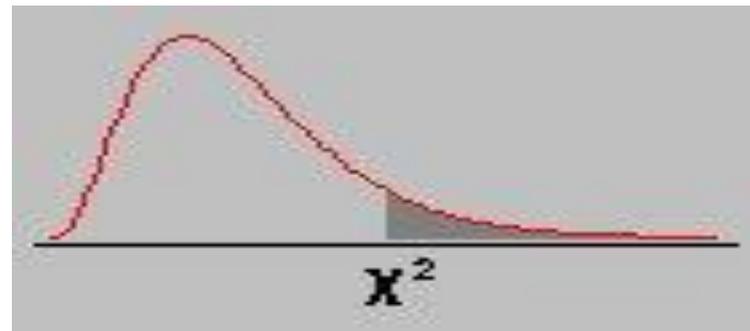
| | | | |
|----|-------|-------|-------|
| 41 | 56.94 | 64.95 | 74.75 |
| 42 | 58.12 | 66.21 | 76.09 |
| 43 | 59.30 | 67.46 | 77.42 |
| 44 | 60.48 | 68.71 | 78.75 |
| 45 | 61.66 | 69.96 | 80.08 |
| 46 | 62.83 | 71.20 | 81.40 |
| 47 | 64.00 | 72.44 | 82.72 |
| 48 | 65.17 | 73.68 | 84.03 |
| 49 | 66.34 | 74.92 | 85.35 |
| 50 | 67.51 | 76.15 | 86.66 |
| 51 | 68.67 | 77.39 | 87.97 |
| 52 | 69.83 | 78.62 | 89.27 |
| 53 | 70.99 | 79.84 | 90.57 |
| 54 | 72.15 | 81.07 | 91.88 |
| 55 | 73.31 | 82.29 | 93.17 |
| 56 | 74.47 | 83.52 | 94.47 |
| 57 | 75.62 | 84.73 | 95.75 |
| 58 | 76.78 | 85.95 | 97.03 |
| 59 | 77.93 | 87.17 | 98.34 |
| 60 | 79.08 | 88.38 | 99.62 |



| | | | |
|----|--------|--------|--------|
| 61 | 80.23 | 89.59 | 100.88 |
| 62 | 81.38 | 90.80 | 102.15 |
| | 82.53 | 92.01 | 103.46 |
| | 83.68 | 93.22 | 104.72 |
| | 84.82 | 94.42 | 105.97 |
| | 85.97 | 95.63 | 107.26 |
| | 87.11 | 96.83 | 108.54 |
| 68 | 88.25 | 98.03 | 109.79 |
| 69 | 89.39 | 99.23 | 111.06 |
| 70 | 90.53 | 100.42 | 112.31 |
| 71 | 91.67 | 101.62 | 113.56 |
| 72 | 92.81 | 102.82 | 114.84 |
| 73 | 93.95 | 104.01 | 116.08 |
| 74 | 95.08 | 105.20 | 117.35 |
| 75 | 96.22 | 106.39 | 118.60 |
| 76 | 97.35 | 107.58 | 119.85 |
| 77 | 98.49 | 108.77 | 121.11 |
| 78 | 99.62 | 109.96 | 122.36 |
| 79 | 100.75 | 111.15 | 123.60 |
| 80 | 101.88 | 112.33 | 124.84 |

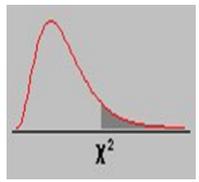
| | | | |
|----|--------|--------|--------|
| 81 | 103.01 | 113.51 | 126.09 |
| 82 | 104.14 | 114.70 | 127.33 |
| 83 | 105.27 | 115.88 | 128.57 |
| 84 | 106.40 | 117.06 | 129.80 |
| 85 | 107.52 | 118.24 | 131.04 |

| | | | |
|----|--------|--------|--------|
| 86 | 108.65 | 119.41 | 132.28 |
| 87 | 109.77 | 120.59 | 133.51 |
| 88 | 110.90 | 121.77 | 134.74 |
| 89 | 112.02 | 122.94 | 135.96 |
| 90 | 113.15 | 124.12 | 137.19 |
| 91 | 114.27 | 125.29 | 138.45 |
| 92 | 115.39 | 126.46 | 139.66 |
| 93 | 116.51 | 127.63 | 140.90 |



| | | | |
|-----|--------|--------|--------|
| 93 | 116.51 | 127.63 | 140.90 |
| 94 | 117.63 | 128.80 | 142.12 |
| 95 | 118.75 | 129.97 | 143.32 |
| 96 | 119.87 | 131.14 | 144.55 |
| 97 | 120.99 | 132.31 | 145.78 |
| 98 | 122.11 | 133.47 | 146.99 |
| 99 | 123.23 | 134.64 | 148.21 |
| 100 | 124.34 | 135.81 | 149.48 |

Example



A sample of 671 diseased person were subjected to treatment, 354 individuals of them, were given drug A. Of those given drug A only 240 patients were survived. On the other hand only 212 patients *who's given drug B were survived* can we conclude that the effectiveness of treatment differ between two drugs (A&B) ????. Let α 0.05

| Out come | Drug A | Drug B | Total |
|----------|---------|---------|---------|
| Survived | 240 | 212 | ????? |
| Died | ??????? | ????? | ??????? |
| Total | 354 | ??????? | 671 |

(also known as a cross tabulation or crosstab)

An important thing is the type of the variable concerned.

| Out come | Drug A | Drug B | Total |
|----------|--------|--------|-------|
| Survived | 240 | 212 | 452 |
| Died | 114 | 105 | 219 |
| Total | 354 | 317 | 671 |

$$H_0 = P_1 = P_2 = P_0$$

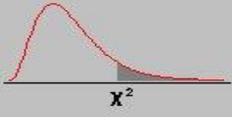
$$H_A = P_1 \neq P_2 \neq P_0$$

We would like to see if there is a

significance difference in the survival rate between the two drugs . Let α 0.05

$$\text{Total Survival rate} = \frac{452}{671} \times 100 = 67.4 \%$$

An important thing is the type of the variable concerned.



Survival rate for A $= \frac{240}{354} \times 100 = 67.8\%$

$$H_0 = P_1 = P_2 = P_0$$

Survival rate for B $= \frac{212}{317} \times 100 = 66.9\%$

$$H_A = P_1 \neq P_2 \neq P_0$$

There is an **observed difference** in the **survival** rate between drug **A** (67.8%) and **B** (66.9%) .

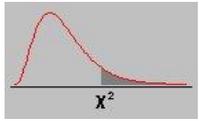
Is this difference in survival rate due to

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- Drug Effectiveness .
- Chance Factor .

| Out come | Drug A | Drug B | Total |
|----------|-------------|------------|------------|
| Survived | 240 (67.5%) | 212(66.9%) | 452(67.4%) |
| Died | 114 | 105 | 219 |
| Total | 354 | 317 | 671 |

Data



Data consist of sample of patients divided into two groups, group A and group B .

Survival rate in group treated by drug **A** was **67.8 %**, and
Survival rate in group treated by drug B was **66.8% .**

Assumption

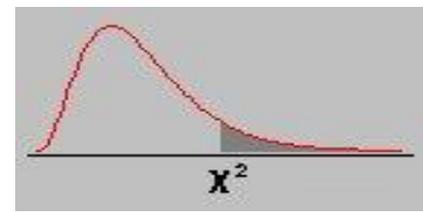
Two independent group of patients given **two different type of treatment** chosen **randomly** from **normal distribution** population .

Formulation of Hypothesis

Ho **????????????????**

HA **????????????????**

Formulation of Hypothesis



Ho

There is **no significance** difference in the **proportion (rate)** of survival between two groups .

survival rate group treated by drug **A** was **67.8%** &
survival rate group treated by drug **B** was **66.9%**

There is **no significance association** between survival rate and **type of treatment** .

$$P1 = P2 = P0 : 67.8\% = 66.9\% = 67.4\%$$

HA

There is a **significance difference** in the survival **rate** between two type of treatment .

$$P1 \neq P2 \neq P0 .: 67.8\% \neq 66.9\% \neq 67.4\%$$

Survival rate is **higher among** group of patients treated by **drug A** .

Critical region

Level of significance 0.95, $\alpha = 0.05$

d.F =

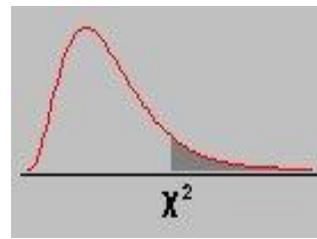
(No. of rows - 1) (No. of column - 1)

$$= (r - 1) (c - 1)$$

$$(2 - 1) (2 - 1) = 1$$

tabulated χ^2 of d.F = 1 with α 0.05

$$= 3.841$$



| Outcome | Drug A | Drug B | Total |
|--------------|------------|------------|------------|
| Survived | 240 | 212 | 452 |
| Died | 114 | 105 | 219 |
| Total | 354 | 317 | 671 |

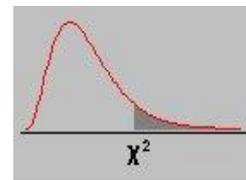
Proper test

Chi Square χ^2

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$E = \frac{\text{total column} \times \text{total rows}}{\text{Grand total}} \quad \text{for each cell}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



$$E_{240} = \frac{354 \times 452}{671} = 238.5$$

$$E_{114} = \frac{354 \times 219}{671} = 115.5$$

$$E_{212} = \frac{452 \times 317}{671} = 213.5$$

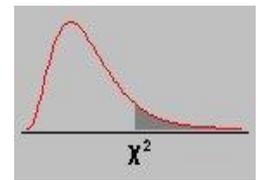
$$E_{105} = \frac{317 \times 219}{671} = 103.5$$

| Outcome | Drug A | | Drug B | | Total |
|-----------------|------------|-------|------------|-------|------------|
| | O | E | O | E | |
| Survived | 240 | 238.5 | 212 | 213.5 | 452 |
| Died | 114 | 115.5 | 105 | 103.5 | 219 |
| Total | 354 | | 317 | | 671 |

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

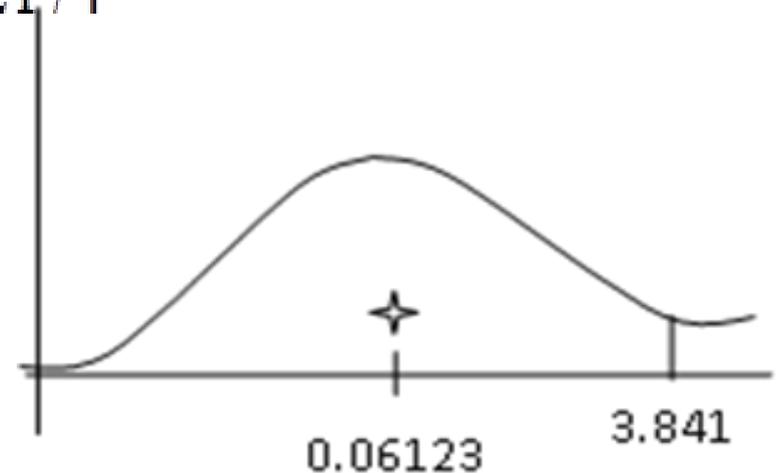


$$= \frac{(240 - 238.5)^2}{238.5} + \frac{(114 - 115.5)^2}{115.5} + \frac{(212 - 213.5)^2}{213.5} + \frac{(105 - 103.5)^2}{103.5}$$

$$= \frac{(1.5)^2}{238.5} + \frac{(1.5)^2}{115.5} + \frac{(-1.5)^2}{213.5} + \frac{(1.5)^2}{103.5} = \frac{2.25}{238.5} + \frac{2.25}{115.5} + \frac{2.25}{213.5} + \frac{2.25}{103.5}$$

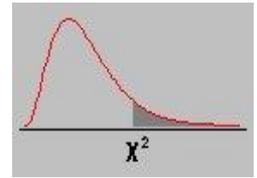
$$= 0.009434 + 0.0195 + 0.01056 + 0.02174$$

$$= 0.061234$$



Calculated χ^2 **fall in** Accept Region \rightarrow **so**

We **not reject** (accept) H_0 .



There is no significance difference in proportion of survival rate between two drugs

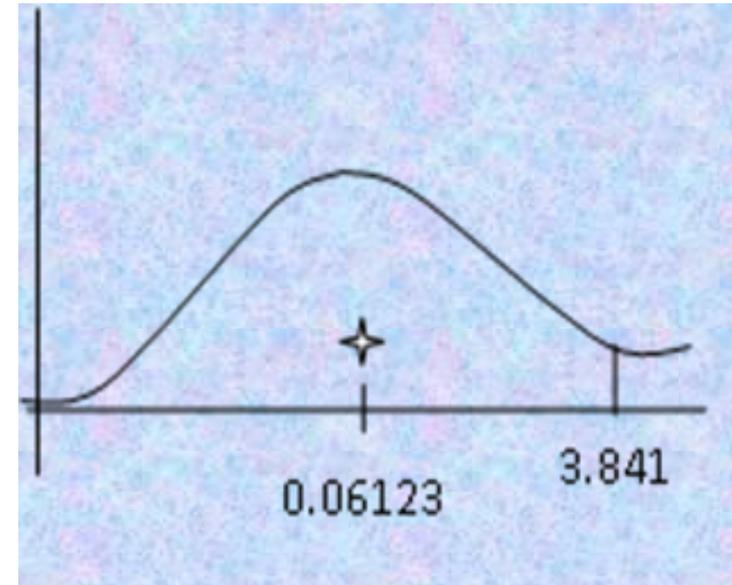
$$P > 0.05$$

Calculated χ^2 less than tabulated χ^2 chance factor increases, influencing factor decrease

There is no significance effect of drug A to increase survival rate .

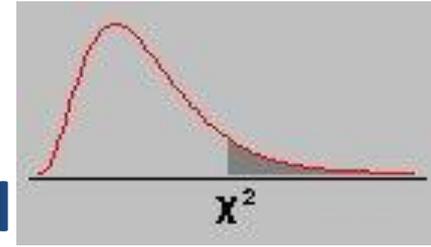
$$P > 0.05$$

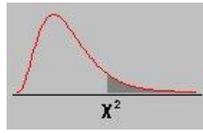
$$P > 0.05 .$$



Chi square calculation procedure

- ✓ Calculate the expected values **E** for each cell
 - ✓ Calculate the value **O - E** for each cell
 - ✓ **O** is the observed
 - ✓ **Square** $(O - E)^2$ **Divide** each squared **O - E** by **E** for each cell
 - ✓ Sum all of the values in previous step
- this result is **called test statistic (calculated χ^2)**
- ✓ identify the **critical chi-square** obtained
 - ✓ from the chi square table.
 - ❑ To **reject the null** hypothesis of equal proportion i.e. of independent variables the value of the **test statistics must exceed** the **critical chi-square** obtained from the chi square table.



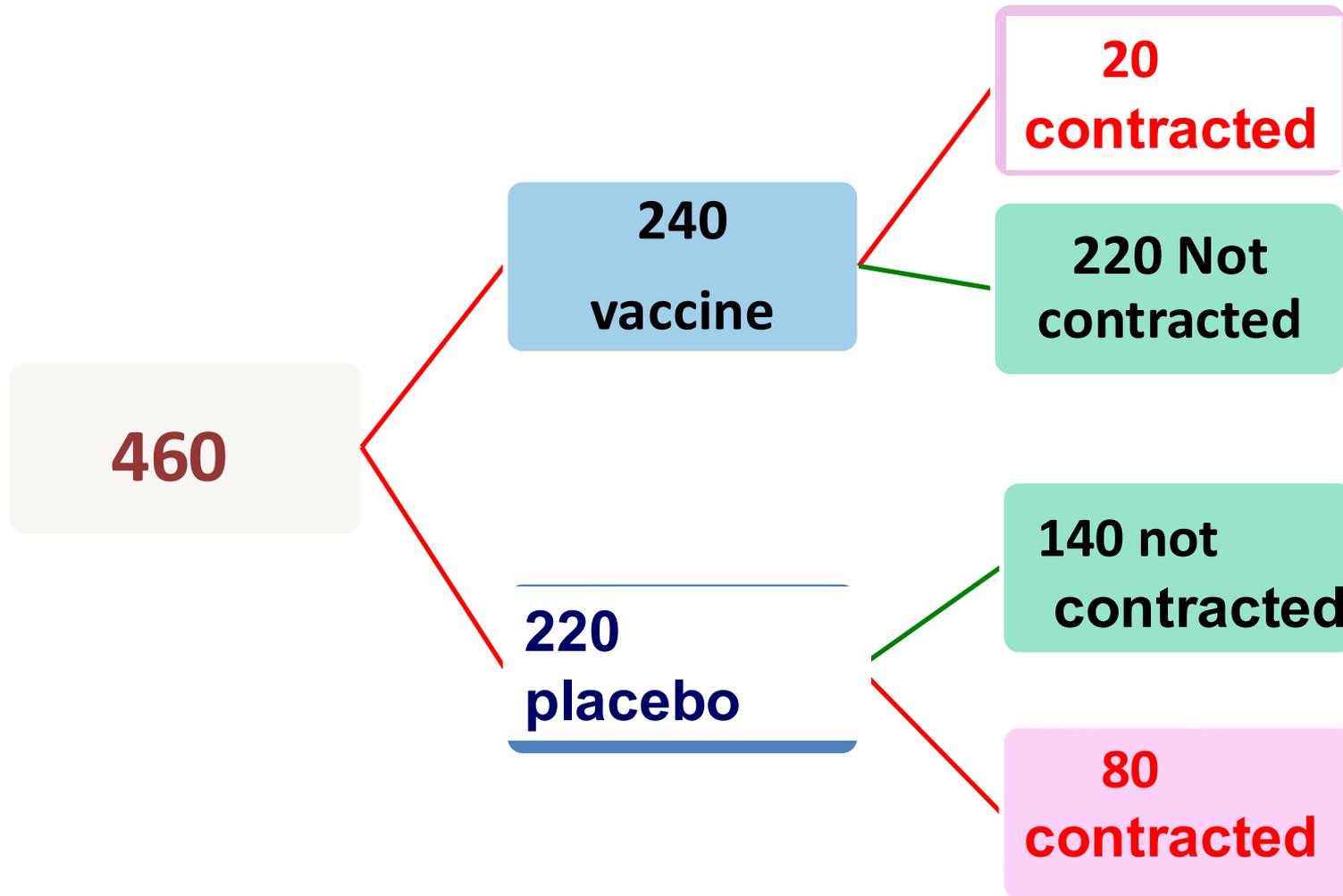


Example 2

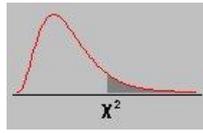
A sample of **460** adult was chosen , **240** were given influenza **vaccine** while the **remaining** given **placebo**
Overall **100** persons contracted influenza, of whom **20** were in vaccine group .

we would like to assess the **strength of evidence** that vaccination **affect the probability** of contracting disease
is there any evidence that **vaccine have an effect** on contracting the disease ??

Total 460 \longrightarrow 100 persons contracted influenza
240 vaccinated \longrightarrow 20 contracted influenza



Total 460 → **100 persons contracted influenza**
 ↓
240 vaccinated → **20 contracted influenza**



We start by display data in 2X2 table .

- The **exposure** is **vaccination** (the row variable) and
- the **outcome** is **contracting influenza** (the column variable)
- we therefore include row % in the table

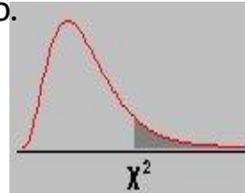
| Exposure | Out come +ve | Out come -ve | total |
|----------|-----------------|-----------------|-------|
| yes | | | |
| no | | | |
| Total | | | |

(also known as a cross tabulation or crosstab)



A sample of 460 adult was chosen , 240 were given influenza vaccine while the remaining given placebo. Overall 100 persons contracted influenza, of whom 20 were in vaccine group .we would like to assess the strength of evidence that vaccination affect the probability of contracting disease .

is there any evidence that vaccine have an effect on contracting the disease ??

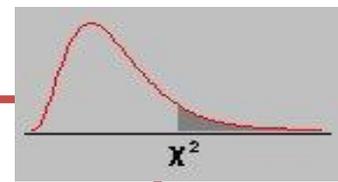


| Given | Contract influenza | | Not contract influenza | | Total |
|---------|--------------------|---|------------------------|---|-------|
| | N | % | N | % | |
| Vaccine | 20 | | 220 | | 240 |
| placebo | 80 | | 140 | | 220 |
| Total | 100 | | 360 | | 460 |

We start by display data in **2X2 table** .

The **exposure** is vaccination (the **row variable**) and the **outcome** is contracting influenza (the **column variable**) we therefore include row % in the table

Total 460 \longrightarrow 100 persons contracted influenza
 240 vaccinated \longrightarrow 20 contracted influenza



| | Contract influenza | | Not contract influenza | | Total |
|---------|--------------------|--------|------------------------|--------|-------|
| | N | (%) | N | (%) | |
| Vaccine | 20 | (8.3) | 220 | (91.7) | 240 |
| placebo | 80 | (36) | 140 | (63.6) | 220 |
| Total | 100 | (21.7) | 360 | (78.3) | 460 |

Overall persons contracting influenza

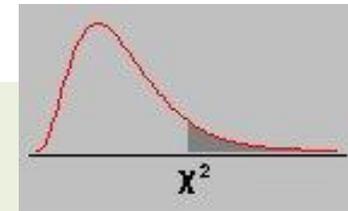
100/460= 21.7%

The chi square compare the **observed** number in each of four categories with the number **expected**

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

E = Total row X total column
Over all total frequency

E expected (E) = $\frac{\text{total column X total row}}{\text{Grand total}}$



$$E_{20} = \frac{240 \times 100}{460}$$

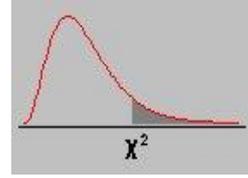
$$E_{220} = \frac{240 \times 360}{460}$$

$$E_{80} = \frac{220 \times 100}{460}$$

| | Contract influenza N (%) | Not contract influenza N (%) | Total |
|---------|--------------------------------|------------------------------------|-------|
| Vaccine | 20 (8.3) | 220 (91.7) | 240 |
| placebo | 80 (36) | 140 (63.6) | 220 |
| Total | 100 (21.7) | 360 (78.3) | 460 |

$$E_{140} = \frac{220 \times 360}{460}$$

$$E \text{ expected (E)} = \frac{\text{total column X total row}}{\text{Grand total}}$$



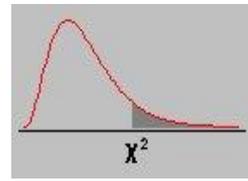
The chi square compare
the **observed** number in each of four categories
with the number **expected**

| | Contract influenza | | Not contract influenza | | total |
|---------|--------------------|------|------------------------|-------|-------|
| | O | E | O | E | |
| Vaccine | 20 | 52.2 | 220 | 187.8 | 240 |
| placebo | 80 | 47.8 | 140 | 172.2 | 220 |
| Total | | 100 | | 360 | 460 |

Then chi square be calculated by calculating **E. frequencies**

if there were no difference in the efficacy between vaccine and placebo.

if the vaccine and placebo having same efficiency then



Then chi square be calculated by calculating **E. frequencies**

we expect to have same proportion in each group that is in the

vaccine group $100/460 \times 240 = 52.2$

in placebo group $100/460 \times 220 = 47.8$

$H_0 = 52.2 = 47.8$

would have contract influenza. .

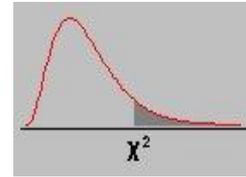
Similarly

$360/460 \times 240 = 187.8$ in vaccine group

$360/460 \times 220 = 172.2$ in placebo group

will escape influenza

$$\chi^2 = \sum \frac{(O - E)^2}{E} \quad \text{d.f.} = 1$$



| | Contracting Influenza | | Not contract influenza | | total |
|---------|-----------------------|------|------------------------|-------|-------|
| | O | E | O | E | |
| Vaccine | 20 | 52.2 | 220 | 187.8 | 240 |
| placebo | 80 | 47.8 | 140 | 172.2 | 220 |
| Total | 100 | | 360 | | 460 |

$$\chi^2 = \frac{(20 - 52.2)^2}{52.2} + \frac{(80 - 47.8)^2}{47.8} + \frac{(220 - 187.8)^2}{187.8} + \frac{(140 - 172.2)^2}{172.2}$$

$$19.86 + 21.69 + 5.52 + 6.02 = 53.99$$



Critical region

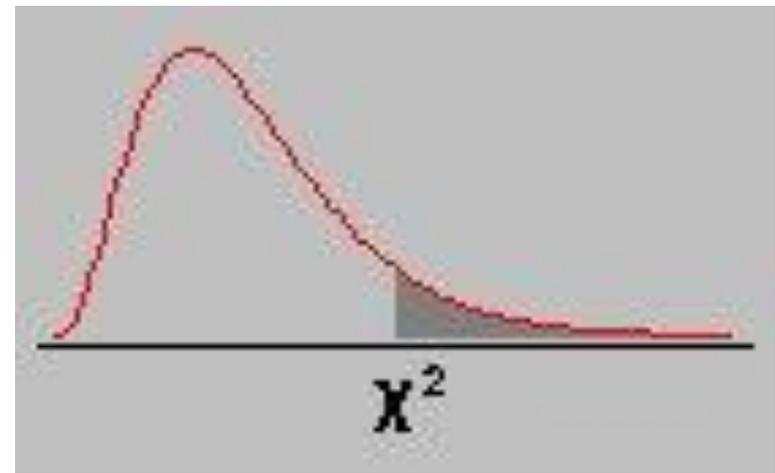
$$d.F = (C - 1) (r - 1)$$
$$= (2 - 1) (2 - 1) = 1$$

$$\alpha = 0.05$$

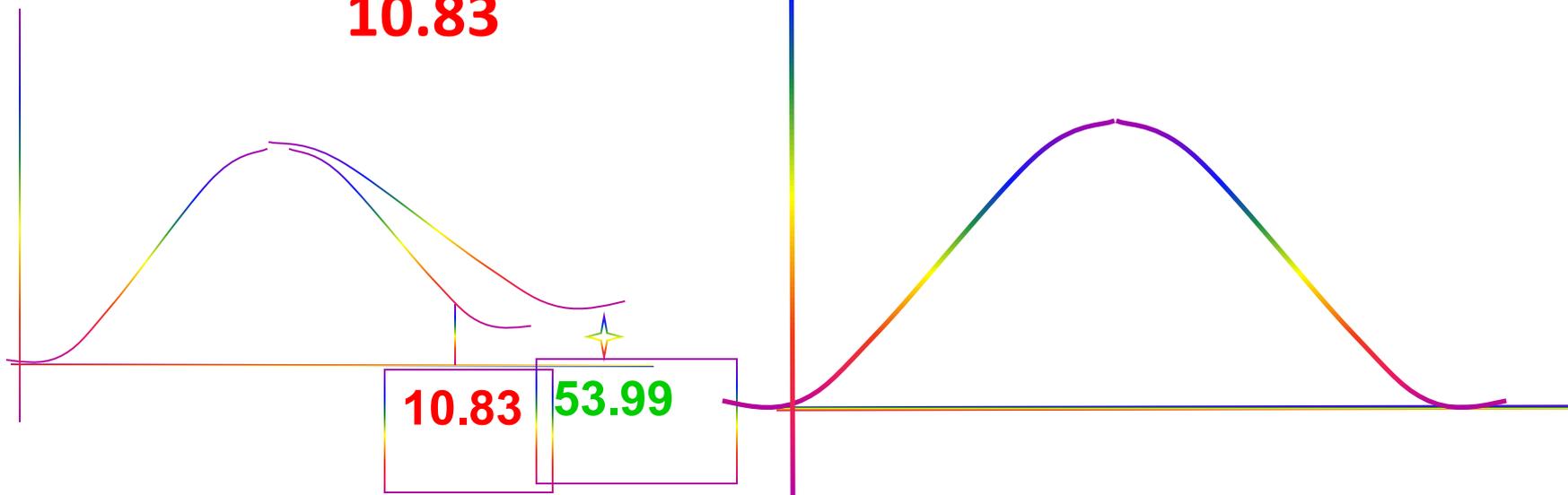
$$\text{tabulated } \chi^2 = 3.84$$

$$6.64$$

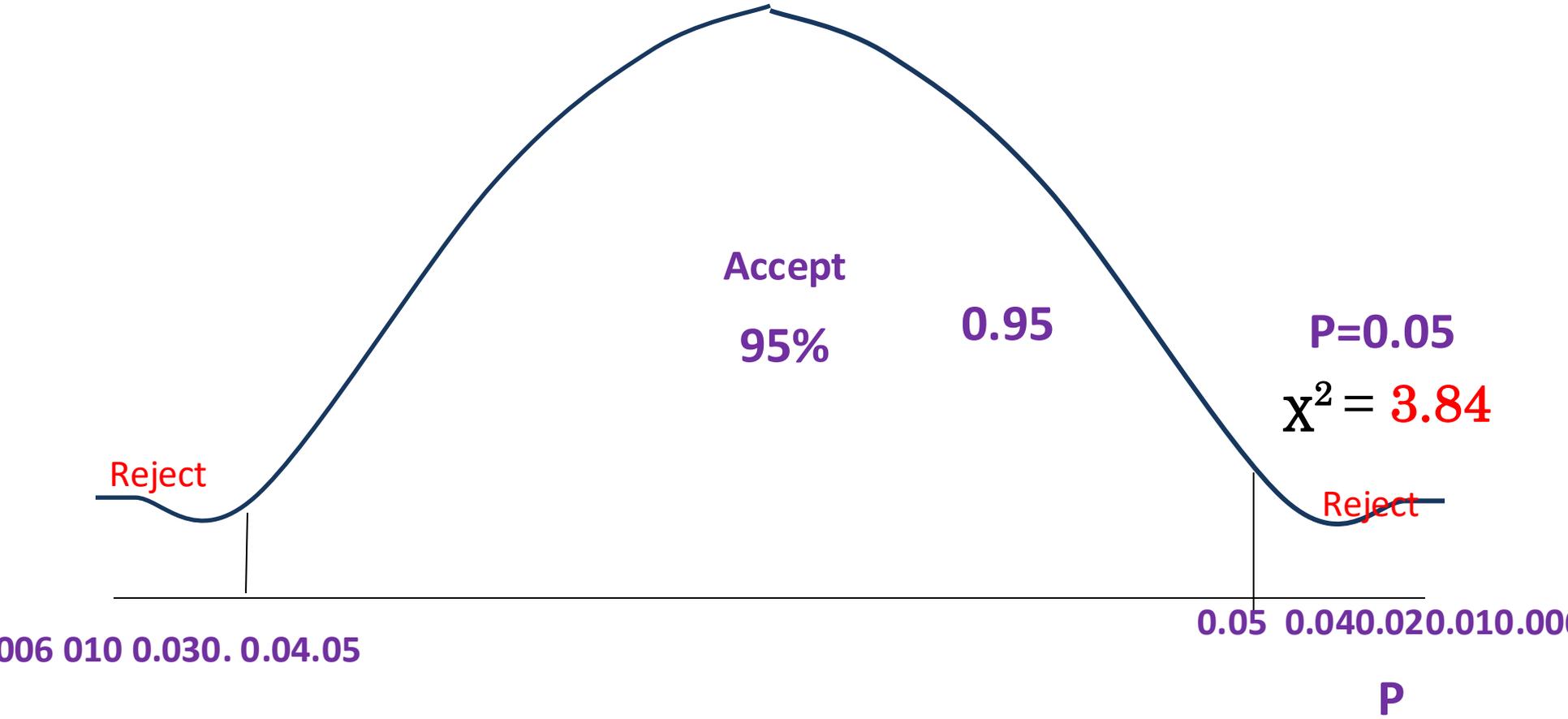
$$10.83$$



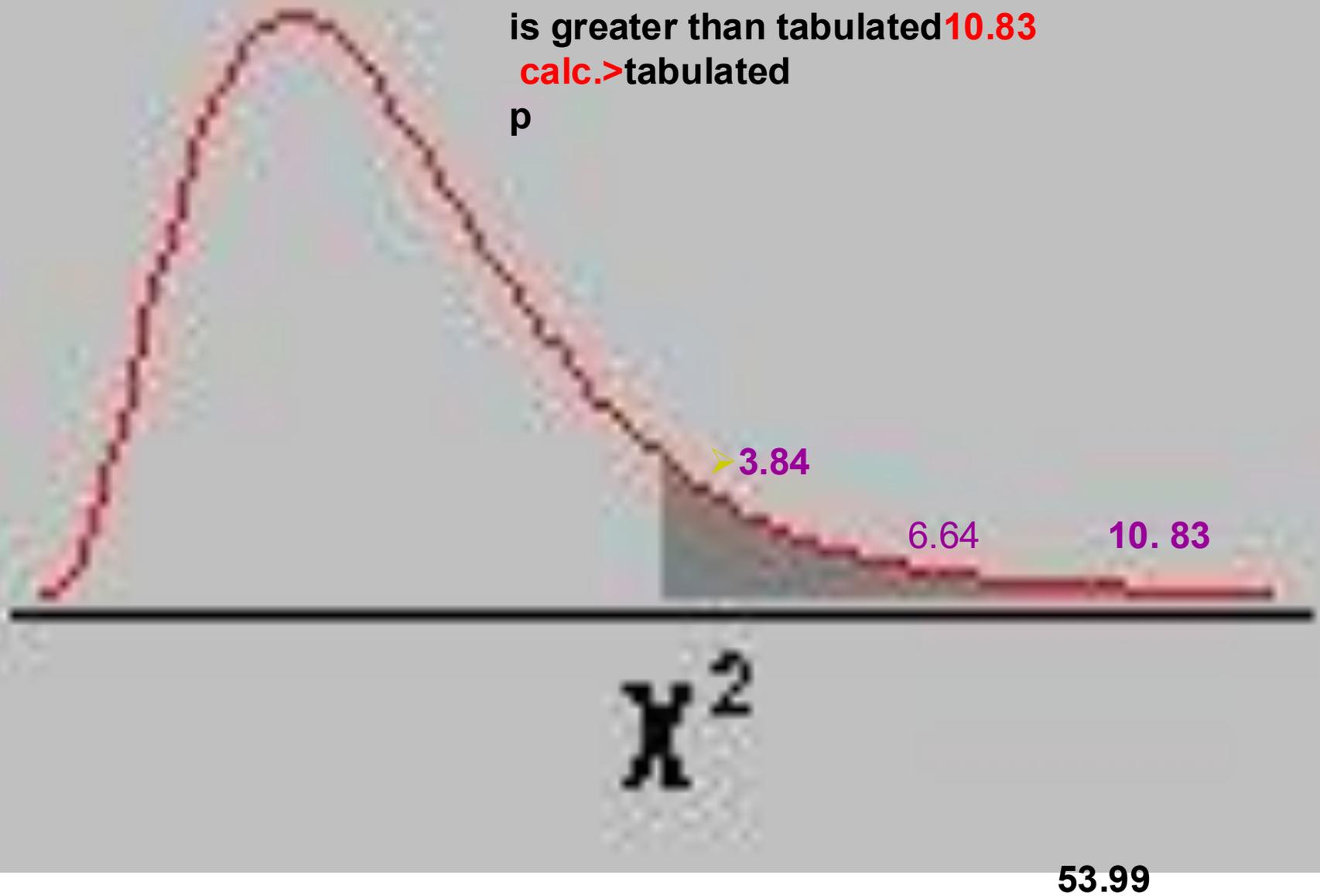
$$10.83$$



your χ^2 53.99



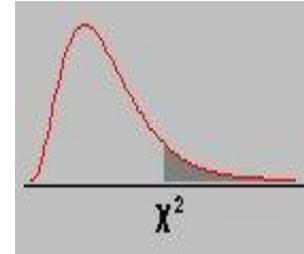
calculated 53.99
is greater than tabulated 10.83
calc.>tabulated
p



p is ??????????

- This mean that
- the probability (p) is less than 0.001
- that chance **factor has effect much less than 0.001**

- and more than 99.999 that this **difference**
- **due to vaccine**

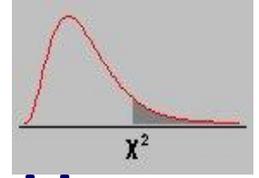


- Thus there is a **strong evidence against** null hypotheses that is saying no effect of vaccine on the probability of contracting influenza .

- there is a **strong evidence** that **vaccine is effective**

- Therefore it is concluded that **vaccine is effective**

Continuity Correction



The chi square test for 2X2 table can be improved by using continuity correction we call it

Yates continuity correction the formula become

$$\chi^2 = \sum \left[\frac{(O - E) - 0.5}{E} \right]^2 \quad \text{d.f.} = 1$$

Pearson's chi-squared test by subtracting 0.5 from the difference between each observed value and its expected value in a 2×2

**Resulting in small value for chi square
(the value of $O - E$) ignoring the sig**

$$X^2 = \frac{(20 - 52.2 - 0.5)^2}{52.2} + \frac{(80 - 47.8 - 0.5)^2}{47.8} +$$

$$\frac{(220 - 187.8 - 0.5)^2}{187.8} + \frac{(140 - 172.2 - 0.5)^2}{172.2}$$

$$X^2 = \frac{1004.89}{52.2} + \frac{1004.89}{47.8} + \frac{100.89}{187.8} + \frac{1004.89}{172.2}$$

$$X^2 = 51.46$$

Thank You

Example III

A sample of 84 mother chosen randomly **20** were smoker who delivered **14** babies with small birth weight (BW) .While the non smoker women delivered **20** small BW babies .can we conclude that maternal smoking has a relation to small birth weight ?

Application of χ^2 .

1. 2×2 table .
2. $a \times b$ table .

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$