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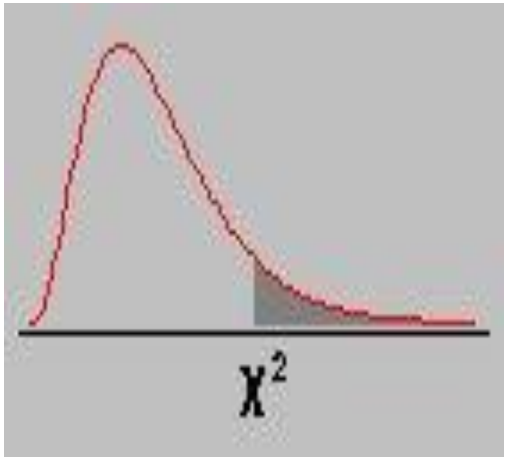


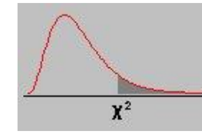
LVII

Chi Square (x^2) test

Part 2

Prof. Dr. Waqar AL-Kubaisy

@ August 5- 2025



Application of χ2

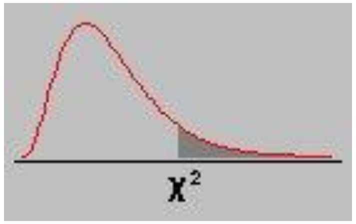
2×2 table .

$r \times c$ table .

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Application of χ2

- 2×2 table
- $r \times c$ table.



Application of $\chi2$

I- 2 × 2 table

The application of χ^2 is to test the significance association between outcome and certain factor that we are interested in .

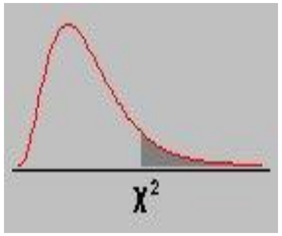
Here we have

two groups with

two outcomes for each group .

In this case we use what we call it 2 × 2 table .

In this case we are going to compare between two proportion of two groups of population .



2×2 table

The application of χ^2 is to test the significance association between outcome and certain factor that we are interested in .

Here we have

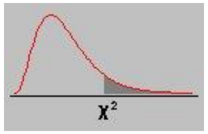
two groups with two outcome for each group

two groups each group with two outcome

In this case we use what we call it 2×2 table .

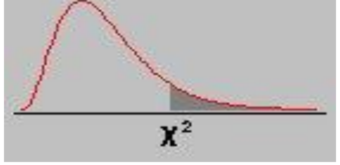
In this case we are going to compare between two proportion of two groups of population .

Example III



A sample of 84 mother chosen randomly **20** were smoker who delivered **14** babies with small birth weight (BW) While the non smoker women delivered **20** small BW babies can we conclude that maternal smoking has a relation to small birth weight ?

mother	Small BW	Normal BW	total
Smoker	14		20
Non smoker	20		
Total			84



<u>Example</u>

A sample of 84 mother chosen randomly 20 were smoker who delivered 14 babies with small birth weight (BW) On the other hand the non smoker women delivered 20 small BW babies .Can we conclude that maternal smoking has a relation to small birth weight ?

	Small BW	Normal BW	total
Smoker	14 (70%)	6 (30%)	20
Non smoker	20 31.3 %)	44 68.7%	64
Total	34 (40.5%)	50	84

Ho ;

small BW and smoking status during pregnancy are not related in the population.

The Two variables are independent

HA:

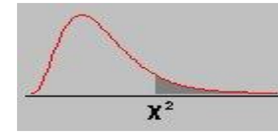
Small BW and smoking status during pregnancy are **related** in the population .

The Two variable are Dependent

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$Ho = P_1 = P_2 = P_0$$

$$H_A = P_1 \neq P_2 \neq P_0$$



If the two variables are unrelated(H0)

then there is no reason why the proportion of small BW among smokers should be different to the proportion of small BW among non smokers mothers (H0)

In another ward these two proportions should be equal

P1 =P2 70% = 31.3%

this difference could be due to chance (H0)

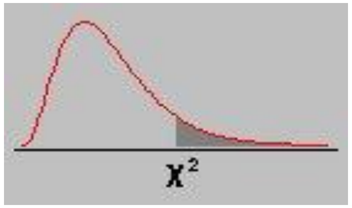
	Small BW		Normal BW		total
Smoker	14	**70%**	6	**30%**	20
Non smoker	20	**31.3 %**	44	**68.7%**	64
Total	34	**40.5%**	50		84

The question is that what proportion would we expect to find if null hypothesis of unrelated variable is true ??

The answer is that

since we got 34 small BW in a total of 84.

34/84= **0.405** **40.5%**



so we expect in smokers group to have ;0.405 x 20=8.1 in nonsmokers 0.405 x 64 = 25.92

An easer way to calculate Expected cell frequency

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Total row X total column

Over all total frequency

$$E14 = \frac{34 \times 20}{84} = 8.094$$

$$E20 = \frac{34 \times 64}{84} = 25.904$$

	Small BW	Normal BW	total
Smoker	14	6	20
Non smoker	20	44	64
Total	34	50	84

Expected freq. = $\dfrac{\text{Total row X total column}}{\text{Over all total frequency}}$

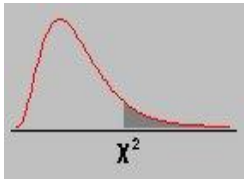
	Small BW		Normal BW		total
	O	**E**	O	**E**	
Smoker	14	**8.1**	6	**11.9**	20
Non smoker	20	**25.1**	44	**30.1**	64
Total	34		50		84

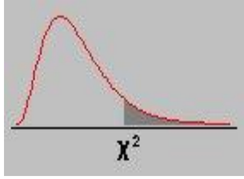
$$\frac{(14-8.1)^2}{8.1} + \frac{(6-11.9)^2}{11.9} + \frac{(20-25.1)^2}{25.1} + \frac{(44-30.1)^2}{30.1}$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

χ^2 = 4,3+2.9+1+6.4 =**14.6**

compare calculated **χ^2 with** tabulated **χ^2**





Critical region

d.F = (C – 1) (r – 1)

= (2 – 1) (2 – 1) = 1

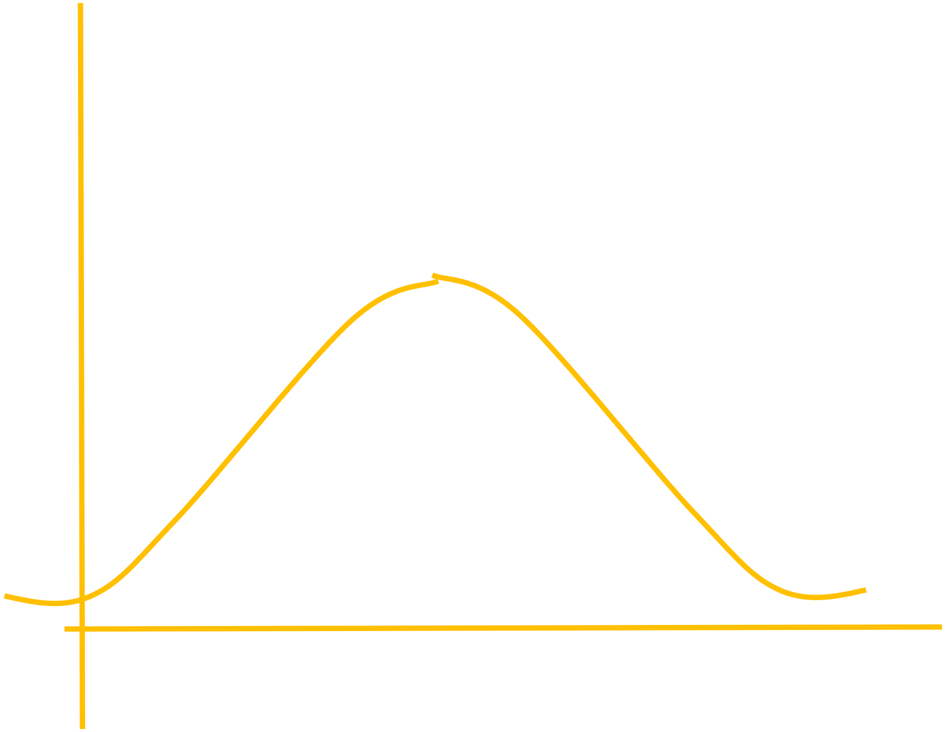
$\alpha = 0.05$

tabulated χ^2 = **3.84**

6.64

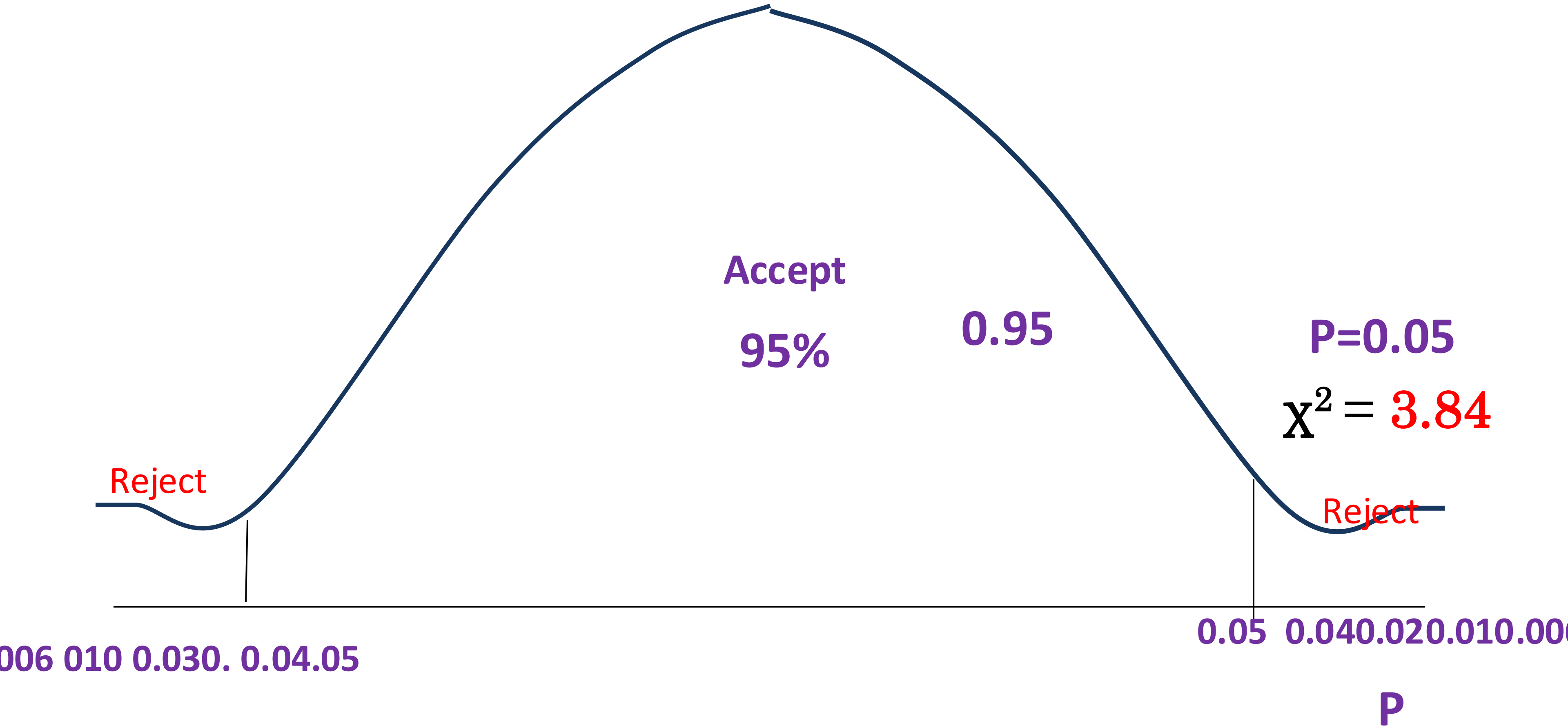
10.83

calculatedχ^2 =**14.6**

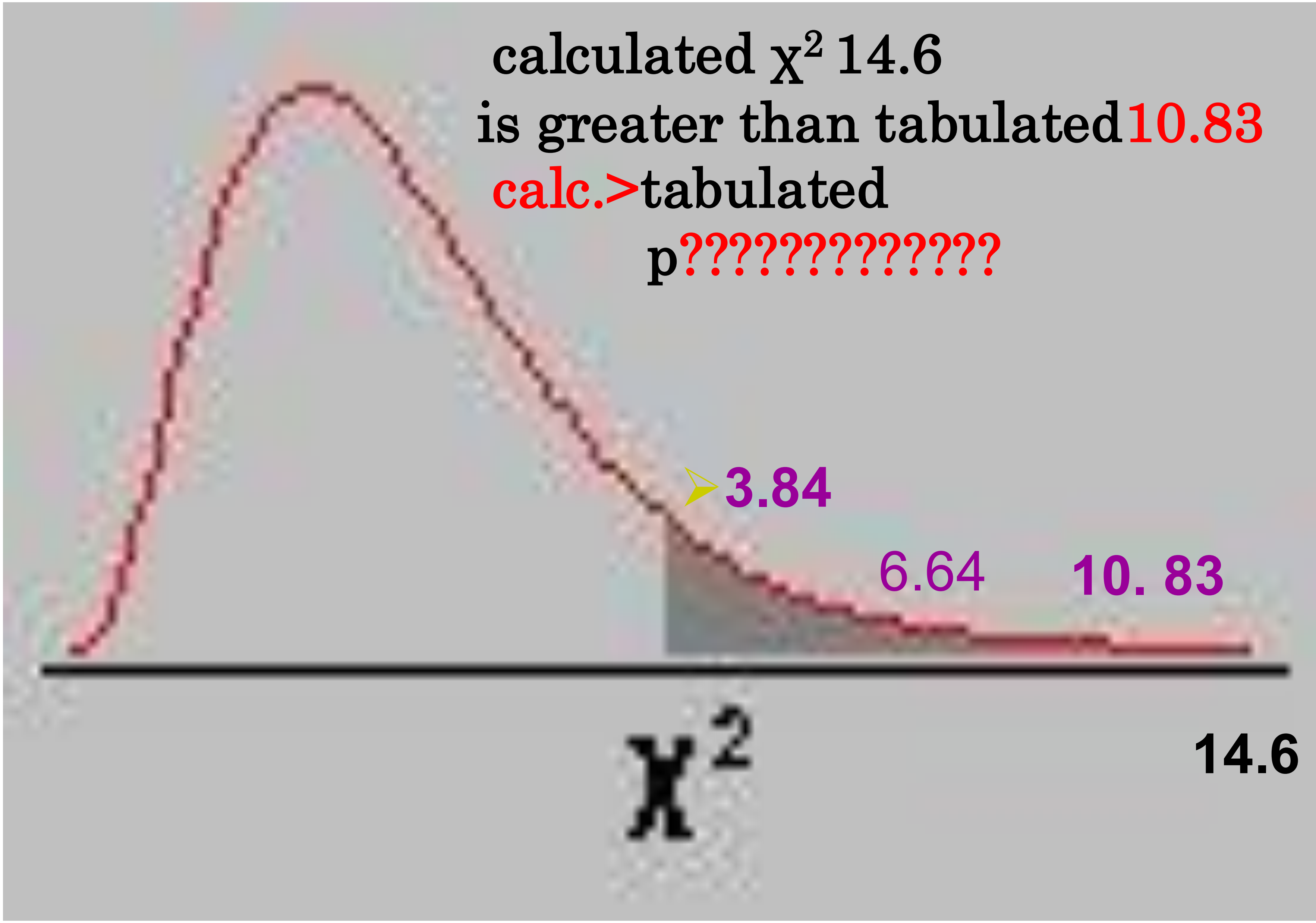


??????

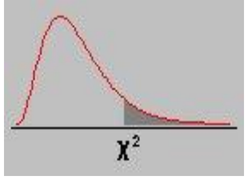
your χ^2 14.6



Your p ?????


calculated χ^2 14.6
is greater than tabulated 10.83
calc.>tabulated
p?????????????
3.84
6.64
10. 83
χ^2
14.6

p is ?????????



- This mean that
- the probability is less than 0.001 that this difference is due to chance factor
  - And more than 99.999 that this difference due to smoking
- Thus there is a strong evidence against null hypotheses that is saying no effect of smoking on the probability of LBW.
  - there is a strong evidence that LBW is related to smoking
- Therefore it is concluded that smoking is risk

p is ?????????

$P > 0.05$	$P > 0.01$	$P > 0.001$
$p < 0.05$	$p < 0.01$	$p < 0.001$

You can answer

if **p-value** associated with chi square is **less than 0.05 or** less than 0.01 → you **reject** null hypoth.

And conclude that

- the two variable are **not independent** or

- there is a **statistically significant difference** in the **proportions**

Application of $\chi 2$

- **2×2 table**
- **$r \times c$ table.**

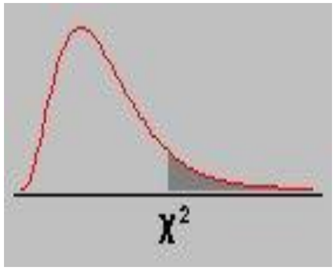
II- r × c

- other application of χ^2 is $r \times c$. , ax b

We have two or more than two groups and or with two or more than two outcome

A contingency table also used

These large table we call it r x c, ax b

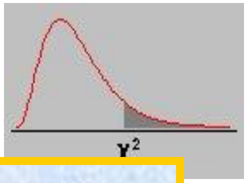


r denotes the numbers of **rows** in the table and **c** the numbers of **columns** .

- **more than two rows** and or **more than two columns .**
- In another word **more than four cells,** **we could have 6, 8, 10**, ...

- Here we have **more than two rows or two columns** .
- We have **two or more groups**
- with **more than two outcome**
- In another word we have more than four cells, we could have 6, 8, 10,

II- r × c



$$X^2 = \sum \frac{(O - E)^2}{E}$$

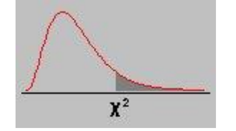
d.f. = (c – 1) (r – 1)

The expected No. must be computed for each cell

$$E = \frac{\text{Column total} \quad \text{X row total}}{\text{Overall total}}$$

There is no continuity correction

- **Chi square is only valid if applied to the**
- **actual numbers in the various categories .**
- **It must never be applied to table showing just proportions or percentages**

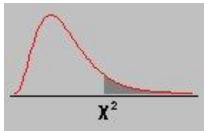


Example

Sample of 273 tuberculosis cases were collected . given three types of treatment either by Para-aminosalicylic acid (**PAS**) **alone or streptomycin alone, or combination of PAS and streptomycin . The outcome of treatment was categorized depen**ding **on the result of sputum exam** either **positive smear positive culture**, **negative smear positive culture** or **negative smear negative culture** .

99 given PAS alone**, 65** of them showed smear +ve &,culture +ve, while only **13 cure**. Of the group (90 patients) who were treated by combination of streptomycin & PAS, **35 were shows negative smear and negative** culture, while **18 of** combined treated patients demonstrated **negative smear & positive** culture .For those treated by streptomycin only , 46 demonstrated smear +ve &,culture +ve ,and **18 negative smear & positive culture**

99 given PAS alone, 65 of them showed smear+ve &,culture +Ve while only 13 cure. Of the group (90 patients) who were treated by combination of streptomycin & PAS 35 were shows negative smear and negative culture while 18 of combined R patients demonstrated negative smear & positive culture .for those treated by streptomycine46 smear +ve &,culture +ve and 18 demonstrated negative smear & positive culture



Type R	+S +C	-S +C	-S -C	Total
PAS	65		13	99
Stre.	46	18		
Com.		18	35	90
Total				273

Type R	+S +C	-S +C	-S -C	Total
PAS	65	21	13	99
Stre.	46	18	20	84
Com.	37	18	35	90
Total	148	57	68	273

cure rate

PAS $\frac{13}{99} \times 100 = 13\%$

Streptomycin $\frac{20}{84}$ X 100 =23.81%

Combine $\frac{35}{90} \times 100 = 39\%$

Failure rat

PAS $\frac{65}{99}$ X 100 =65,7%

Streptomycin $\frac{46}{84}$ X 100 =54.8%

Combine $\frac{37}{90} \times 100 = 41\%$

Total cure rate $\frac{68}{273} \times 100 = 25\%$

Type R	+S +C	-S +C	-S -C	Total
PAS	65	21	13	99
Stre.	46	18	20	84
Com.	37	18	35	90
Total	148	57	68	273

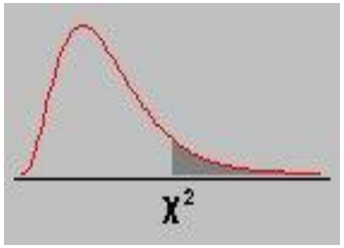
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Data

Qualitative data, No. Of T.B patients, treated by 3 different regime (PAS alone, Streptomycin alone or combine both) . Outcome of treatment categorized into 3 group (Failure, not cure and cure) .

Assumption

Independent random sample chosen from normal distribution population



Formulation of Hypothesis

Ho

There is no significance difference in cure rate among the three different treated group .

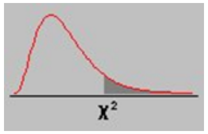
$$P1¬ = P2 = P3 = P0 .$$

The difference observed is due to chance factor, sampling error and sampling variability .

There is no significance difference in cure rate among the three different treated group .
$P1 = P2 = P3 = P0$.
The difference observed is due to chance factor, sampling error and sampling variability .
There is no significance association between cure rate level and type of treatment .

HA

There is a significance difference in cure rate between three group .
This difference due to effect of different treatment . There is no or minimum effect of chance factor .
$P1 \neg \neq P2 \neq P3 \neq P0$



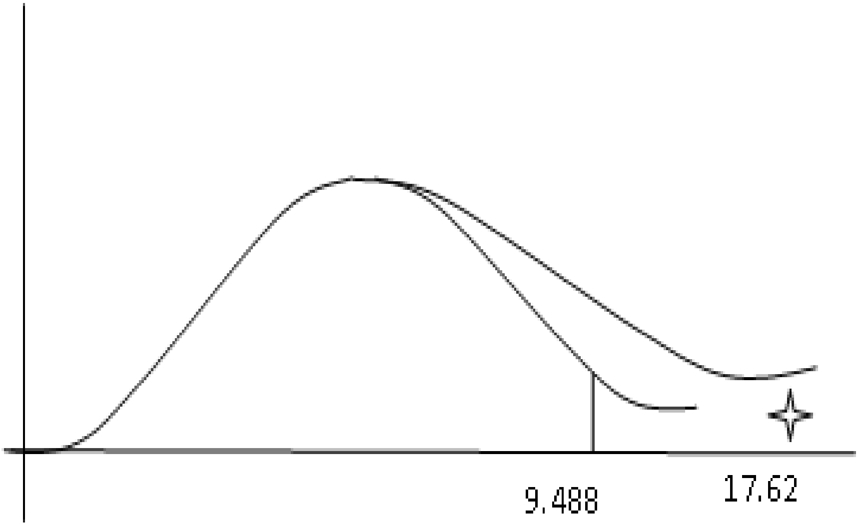
Critical region

d.F = $(C - 1)(r - 1)$

$= (3 - 1)(3 - 1) = 4$

$\alpha = 0.05$

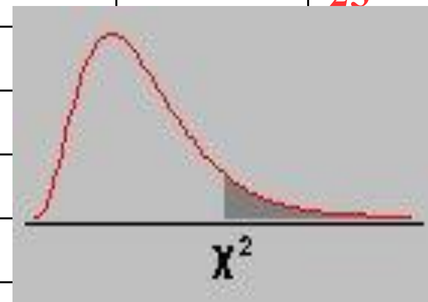
tabulated $\chi 2$ = 9.488

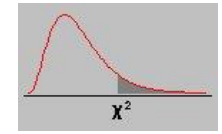


$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Table of Chi-square statistics

df	P =0.05	P = 0.01	P = 0.001
1	3.84	6.64	10.83
2	5.99	9.21	13.82
3	7.82	11.35	16.27
4	9.49	13.28	18.47
5	11.07	15.09	20.52
6	12.59	16.81	22.46
7	14.07	18.48	24.32
8	15.51	20.09	26.13
9	16.92	21.67	27.88
10	18.31	23.21	29.59
11	19.68	24.73	31.26
12	21.03	26.22	32.91
13	22.36	27.69	34.53
14	23.69	29.14	36.12
15	25.00	30.58	37.70
16	26.30	32.00	39.25
17	27.59	33.41	40.79
18	28.87	34.81	42.31
19	30.14	36.19	43.82
20	31.41	37.57	45.32
21	32.67	38.93	46.80
22	33.92	40.29	48.27
23	35.17	41.64	49.73
	36.42	42.98	51.18
	37.65	44.31	52.62
	38.89	45.64	54.05
	40.11	46.96	55.48
28	41.34	48.28	56.89
29	42.56	49.59	58.30
30	43.77	50.89	59.70
31	44.99	52.19	61.10
32	46.19	53.49	62.49
33	47.40	54.78	63.87
34	48.60	56.06	65.25
35	49.80	57.34	66.62
36	51.00	58.62	67.99
37	52.19	59.89	69.35
38	53.38	61.16	70.71
39	54.57	62.43	72.06
40	55.76	63.69	73.41





E expected (E) $= \dfrac{\text{total column X total row}}{\text{Grand total}}$

E65 $= \dfrac{99 \text{ X}148}{273} = 53.67$

E21 $== \dfrac{99 \text{ X}57}{273} = 20.67$

E13$= = \dfrac{99 \text{ X}68}{273} = 24.66$

E46$= \dfrac{84 \text{ X}148}{273} = 45.54$

E18$= \dfrac{84 \text{ X}57}{273} = 17.54$

E20$= \dfrac{84 \text{ X}68}{273} = 20.9$

E37$= \dfrac{90 \text{ X}148}{273} = 48.8$

E18$= \dfrac{90 \text{ X}57}{273} = 18.8$

E35$= \dfrac{90 \text{ X}68}{273} = 22.42$

Type R	+S +C	-S +C	-S -C	Total
PAS	65	21	13	99
Stre.	46	18	20	84
Com.	37	18	35	90
Total	148	57	68	273

$$\chi^2=\sum\frac{(O-E)^2}{E}$$

$$\frac{(65-53.67)^2}{53.67}+\frac{(21-20.67)^2}{20.67}+\frac{(13-24.66)^2}{24.66}+\frac{(46-45.54)^2}{45.54}\quad\frac{(18-17.54)^2}{17.54}+$$

$$\frac{(20-20.9)^2}{20.9}+\frac{(37-48.8)^2}{48.8}+\frac{(18-18.8)^2}{18.8}+\frac{(35-22.42)^2}{22.42}$$

=2.4+0.005+5.513+0.005+0.012+0.047+2.85+0.034+7.067= 17.978

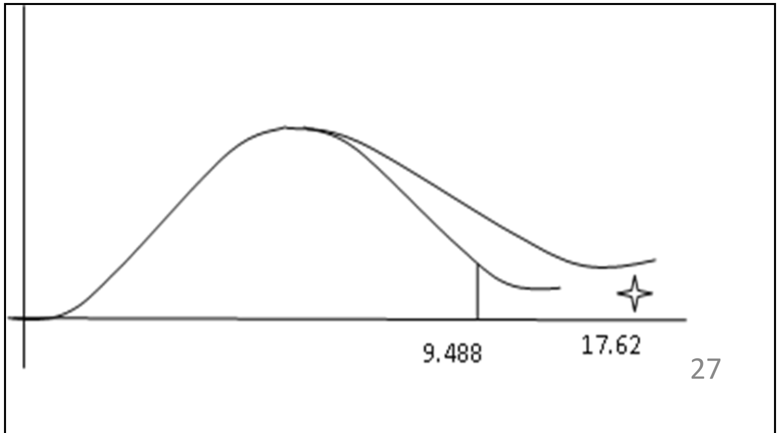
Calculated χ^2 greater than **tabulated χ^2 .**

Calculated χ^2 fall in area of **rejection**,

so we **reject Ho** = we **reject no significance difference** .

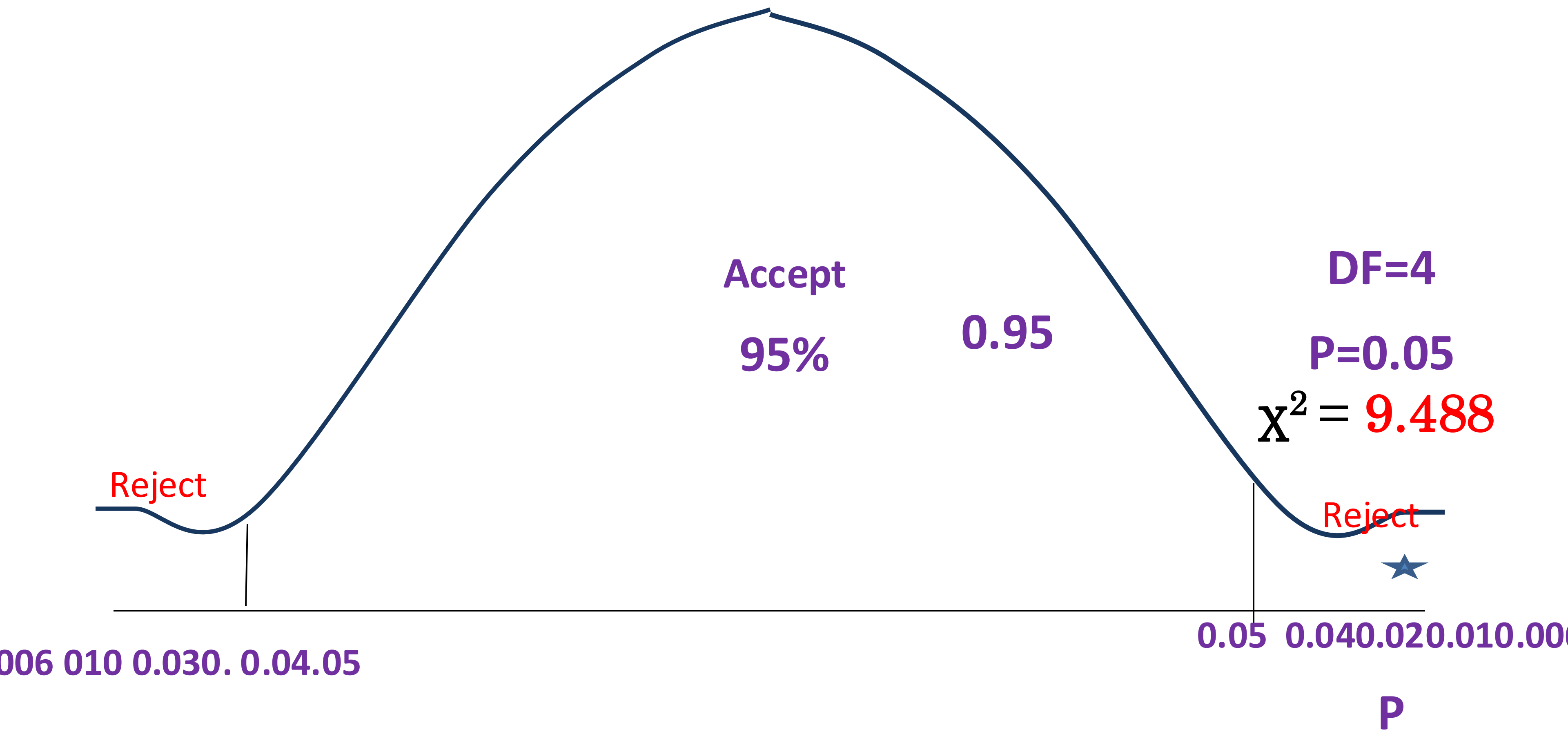
There is significance difference in cure rate between the three groups . P ??????

P < 0.05



your χ^2 17.978

Test statistics =17.978

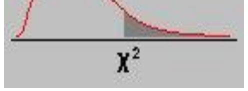


17.978 fall in ????

	Total	succeeded	%	Not succeeded
Baghdad	220	180	82%	40
UiTM	200	170	85%	30
Syria	320	200	62.5%	120
Mutah	380	220	57.9%	160
	1120	770		350

$$770/1120 = 0.687$$

$$\boxed{770/1120 \times 100 = 68.7\%}$$



Data

Qualitative data consist of sample of medical students divided into four groups,.

Variation in the Successful rate was detected

Assumption

.

Formulation of Hypothesis

Ho

HA

Expected freq. = $\frac{\text{Total row X total column}}{\text{Over all total frequency}}$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

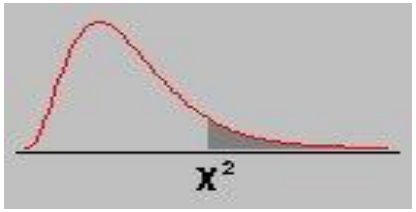
	Succeeded		Not succeeded		Total
	O	E	O	E	
Baghdad	180	151.25	40	68.75	220
UiTM	170	137.5	30	62.5	200
Syria	200	220	120	100	320
Mutah	220	261.25	160	118.75	380
Total	770		350		1120

Example III

In a study to determine whether there is an association between type of therapy and outcome in mental illnesses. A group of 120 mentally ill patients were given drug alone and another group of 120 mentally ill patients were given drug and psychotherapy.

The outcome of all patients were assigned as : **Deteriorated**, **unchanged** or Improved. In the first group, 6 patients were deteriorated and only 49patients were improved, while the remaining were unchanged. On the other hand in the second group, 31 patients were unchanged and 78 patients were improved. At 0.95 level of significance do these data provide evidence to indicate that psychotherapy is an effective measure in treating mentally ill patients?

Treatment	improved	No change	Deteriorated	Total
medication	49	65	6	120
Medication + psychotherapy	78	31	**11**	120
Total	127	96	17	240



Validity of χ^2

When the expected numbers are very small the chi square test is not good enough

We recommended other test (Exact Test)

Thus χ^2 is valid

- when the overall total is more than 40 , regardless the expected values

and

- when the overall total between 20 and 40 provided that all expected values are at least 5

Validity of Chi Square

χ^2 is valid

- when the overall total is more than 40 , regardless the expected values and
- when the overall total between 20 and 40 provided that all expected values **are at least 5**
- Chi square is valid provided that
- less than 20% of expected numbers **are** less than 5
- And **none is less than 1**

- When the expected numbers are very small the chi We recommended other test (**Fisher's exact test**
- Chi square test is not valid **when we have cell zero**

This restriction can be overcome by

combining rows or columns with the low expected numbers provide that these combination make biological sense

Fisher's exact test

is a statistical significance test used in the analysis of contingency tables where sample **sizes are small**.

It is named after its inventor, R. A. Fisher **Sir Ronald Aylmer Fisher**

The test is useful for categorical data that result from **classifying** objects in **two different ways**;

it is used to examine the **significance** **o**f the **association** (contingency) between the **two kinds of classification**

Fisher's exact test is more accurate than the chi-squared test of independence **when the expected numbers are small**.

Most uses of the Fisher test involve, like this example, a 2 × 2 contingency table.

With large samples, a chi-squared test can be used in this situation

When to use **Fisher's exact** test

Fisher's exact test is used when you have

two nominal variables.

A data set in rows and columns.

Fisher's exact test is more accurate than the chi-squared test of independence **when the expected numbers are small**.

The most common use of **Fisher's** exact test

is for 2×2 tables,

You can do Fisher's exact test for greater than two rows and columns.

You can do Fisher's exact test for greater than two rows and columns.

- **If all of the expected values are very large,**
Fisher's exact test becomes **computationally impractical**;

in case the total number of observations is less than 20),
you should not perform the *Chi-square test* but you should use *Fisher's exact test*.

Thank You